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**ANALYTICAL APPROXIMATIONS
VOLUME 13**

**Cecil Hastings, Jr.
James P. Wong, Jr.**

P-441

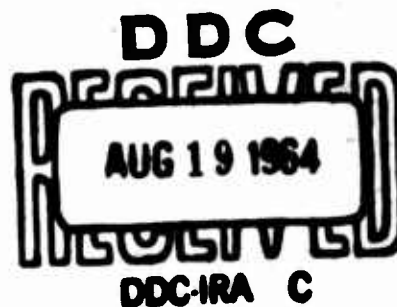
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9 October 1953

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Analytical Approximation

Unnamed Definite Integral: To better than .00055
over $(0, \infty)$,

$$N(x) = \frac{30}{\pi^4} \int_0^{\infty} \frac{e^{-\left(\frac{x}{t}\right)^7} t^7 dt}{e^{t^2} - 1}$$

$$= \frac{1}{1 + .30382x^6 - .55605x^7 + .34791x^8 - .10369x^9 + .01245x^{10}}.$$

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Analytical Approximation

Pearson Cosine Transformation: To better than
.00017 over (0,1),

$$r(x) = \cos \left(\frac{\pi}{1 + \sqrt{x}} \right)$$

$$\approx \frac{-1 - 4.828\eta + 7.866\eta^2 - 2.038\eta^3}{1 + 5.560\eta - 4.985\eta^2 + .385\eta^3}, \quad \eta = \frac{x}{.16 + .84x}$$

$r(x^{-1}) = -r(x)$ can be used to obtain function values
over (1,∞).

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Analytical Approximation

Bessel Function: To better than .00008 over $(0, \infty)$,

$$e^{-x} I_1(x) \doteq \frac{x}{\sqrt[4]{15.4 + 74.8x + 67.2x^2 + 235.8x^3 + 43.5x^4 + 59.4x^5 + 39.6x^6}}.$$

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Analytical Approximation

Mach Number in Terms of Pressure Ratio: To better than .0021
over $.3 \leq M \leq 3$, the inverse of the function defined by

$$x = \frac{P_S}{P_R} = \left[1 + \left(\frac{\gamma-1}{2} \right) M^2 \right]^{-\frac{\gamma}{\gamma-1}}$$

over $.3 \leq M \leq 1$ and

$$x = \frac{P_S}{P_R} = \frac{\left[\left(\frac{2\gamma}{\gamma+1} \right) M^2 - \left(\frac{\gamma-1}{\gamma+1} \right) \right]^{\frac{1}{\gamma-1}}}{\left[\left(\frac{\gamma+1}{2} \right) M^2 \right]^{\frac{\gamma}{\gamma-1}}}$$

over $1 \leq M \leq 3$ where $\gamma = 1.4$, is given by

$$M = \frac{8.11 + 23.60x - 30.66x^2 + 8.98x^3}{1 + 28.70x - 15.99x^2 - 5.74x^3}.$$

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Analytical Approximation

Natural Addition Logarithm: To better than .00026
over $0 \leq x \leq \infty$,

$$\ln(1+e^{-x}) \doteq \frac{\ln 2}{(1+.3581x+.1151x^2+.0094x^3+.0052x^4)^2}.$$

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Analytical Approximation

Natural Addition Logarithm: To better than
.000,045 over $0 \leq x \leq \infty$.

$$\ln(1+e^{-x}) \doteq \frac{\ln 2}{(1+.36123x+.10204x^2+.02411x^3-.00055x^4+.00069x^5)^2}.$$

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Analytical Approximation

Natural Addition Logarithm: To better than .000,008 over $0 \leq x \leq \infty$,

$$\ln(1+e^{-x}) \doteq \frac{\ln 2}{(1+.360571x+.105546x^2+.018760x^3+.002654x^4-.000100x^5+.000066x^6)^2}.$$

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